

ON THE DICTIONARY BETWEEN ERGODIC TRANSFORMATIONS,  
KRIEGER FACTORS AND ERGODIC FLOWS

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Krieger's theorem sets up a one to one correspondence between orbit equivalence classes of ergodic transformations, isomorphism classes of Krieger factors and conjugacy classes of ergodic flows. The theorem suggests the possibility of finding a correspondence between the properties of the three categories of objects. This thesis is concerned with this task, which is analogous to compiling a dictionary between the objects involved.

The correspondence of Krieger is set up by maps between the categories, one of which (from transformations to flows) is the stable range map. In Chapter 1, the inverse of the stable range map is constructed. Thus given a measured flow  $F$ , there is an explicit way of constructing a transformation whose stable range is  $F$ . This provides two way communication between the category of transformations and the category of flows. Many results in the thesis require proofs using this construction.

Chapter 2 investigates the entry "finite invariant measure" on flows. A flow  $F_t$  on a standard measure space  $(\Omega, \nu)$  is said to admit a finite invariant measure if there exists a finite measure  $\mu \sim \nu$  such that  $\mu \circ F_t = \mu$  for all  $t$ . It is found that this corresponds to the property

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of a Krieger factor containing a  $III_1$  subfactor which is the range of a faithful normal conditional expectation.

In chapter 3 the notion of a  $II_1$ -pair is defined on the category of equivalence relations. The corresponding entries in the other categories are found; for flows, it corresponds to one flow being a factor of the other, with a kind of "relatively invariant" measure between them.

The proofs of the results require the knowledge of ergodic theory and operator algebras. One conclusion of the work is that research in this area is promising and that more knowledge in this area will contribute to the advancement of the fields both of ergodic theory and operator algebras.

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